

## CORRELATION BETWEEN WIND VELOCITIES AT THE SURFACE AND THOSE IN THE FREE AIR.

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## SYNOPSIS.

Correlation coefficients between wind velocities at the surface and those at various heights up to 3,000 meters have been computed from kite records made at Broken Arrow, Okla., Drexel, Nebr., Ellendale, N. Dak., Groesbeck, Tex., and Royal Center, Ind., after first classifying the data by surface wind direction. In all cases where the correlation coefficient showed a significant value of 0.50 or more, with a relatively small probable error, the regression coefficients were determined for computing the velocities aloft from those at the surface.

The residuals for each group (i. e., each surface direction for which a correlation coefficient of 0.50 or more was obtained), when the observed velocities were subtracted from the computed, were found to be practically indistinguishably distributed and were therefore regarded as homogeneous. In view of this, they were all grouped together and a frequency polygon drawn. This exhibited a slight skewness and therefore the normal curve of error was not strictly applicable. This is shown, however, in the diagram. A smooth curve was fitted by inspection, since it was thought that all practical purposes would be served by merely showing graphically the degree of probability of a certain variation in the computed velocity from the actual. The arithmetic mean (disregarding signs) of the residuals was found to be 2.8, the median, zero class, and the mode, +2 class.

## INTRODUCTION.

Increasing interest is being manifested as time goes on in weather conditions in the free air, due principally to the steady advancement being made in aeronautics and the consequent demand for meteorological information. Any possible short and direct methods of determining existing conditions aloft are therefore urgently needed. Considerable progress in various branches of meteorology is being made under this pressure of demand, and the outlook continues encouraging. Much of this progress, however, is limited to our ability to obtain numerous free-air observations, and this ability in turn to the available funds for carrying on the work.

## OBJECT AND METHOD OF THE STUDY.

The determination of the linear correlation between two variables serves as a basis for the least-square computation of one variable in terms of the other. Obviously then, free-air wind velocities can be determined, within limited degrees of accuracy, if a significant degree of correlation exists between them and the velocities at the surface. This principle is being successfully applied in various branches of meteorological investigation, such as in predictions of morning minimum temperatures from the depression of the dew point at the preceding evening observation; and estimates of crop yields from monthly rainfall distributions.<sup>1</sup> So far as known this method has never been used, however, in computing free-air wind velocities from those at the surface.

Coefficients of correlation for surface and upper-wind velocities have, however, been computed from a comparatively small number of observations made at Avesnes le Comte and St. Omer in France. These values were close to 0.75 (the probable errors not given) when surface winds were correlated with those at various heights up to 3,000 feet. No classification by wind direction was made in these, however, there being available only some 30 and 60 pilot-balloon observations, respectively.<sup>2</sup>

Many empirical methods have been suggested for computing the velocity aloft but probably none has ever proved of sufficient reliability for practical pur-

poses. Nearly all have been based on only a few observations and when used give exceedingly inaccurate results for individual cases.

Velocities a short distance above the ground may be computed from the formula,<sup>3</sup>

$$V = v \sqrt{\frac{H+22}{h+22}},$$

where  $V$  is the velocity (m. p. s.) of the wind at the height  $H$  (about 16 meters) above ground, computed from the known velocity at the height  $h$  (2 to 8 meters) above ground.

The equation,

$$\frac{V}{v} = \left(\frac{H}{h}\right)^{0.25},$$

was found by Douglas<sup>4</sup> to fit his observations for heights between 100 and 600 meters fairly well.

Shaw<sup>5</sup> suggests as a likely formula,

$$V = \frac{H+a}{a} \times V_0$$

in which  $V$  is the velocity at the height  $H$  above ground,  $V_0$  the observed anemometer velocity, and  $a$  a constant, obviously dependent upon surrounding topography, anemometer exposure, and perhaps other factors. The formula is supposed to apply only until the gradient velocity is reached.

In addition to these and numerous other empirical equations, mention should be made of the expressions obtained by Taylor<sup>6</sup> and others from the hydrodynamical theory of turbulent motion.

The writer has computed the correlation coefficients between velocities at the surface and those at various levels up to 3 kilometers above mean sea-level (M. S. L.) classifying the winds according to surface direction. Those having a significantly high coefficient of correlation (0.50 or greater with a probable error less than one-sixth of the coefficient) were segregated and the regression equations determined from which the upper wind velocity is computed in terms of that at the surface.

Kite records obtained at Broken Arrow, Okla., Drexel, Nebr.; Ellendale, N. Dak.; Groesbeck, Tex.; and Royal Center, Ind., were used and classified by surface wind direction. The four cardinal points, N., E., S., and W., were regarded as sufficient for the purpose but where the number of available observations for these directions was too small to give a reliable coefficient of correlation, the adjacent directions (to 16 points) were included in sufficient quantity to bring the total number up to that necessary to give a dependable value. This is indicated in figure 1 by brackets inclosing the directions which were so grouped.

It has been suggested that possibly a better grouping would have resulted if the records had been classified according to the various quadrants of high and low pressure areas in which they were made. This may be true, but such a classification would require a much greater number of records since only those in well-developed highs and lows could be used. Furthermore, the quadrant classification would defeat the essen-

<sup>1</sup> Stevenson, *Jr. Scot. Metr. Soc.*, p. 348-1880.

<sup>2</sup> *Nature*, 33, p. 593, 1885.

<sup>3</sup> *Adv. Com. for Aero., R. & M. No. 9*, p. 8, 1909.

<sup>4</sup> *Phil. Trans.*, V. 215, p. 1, 1915.

<sup>5</sup> Smith, J. Warren. *MO. WEATHER REV., SUPPLEMENT No. 16*, 1920.

<sup>6</sup> *The Computer's Handbook*, M. O. 223, Section V, p. 67.

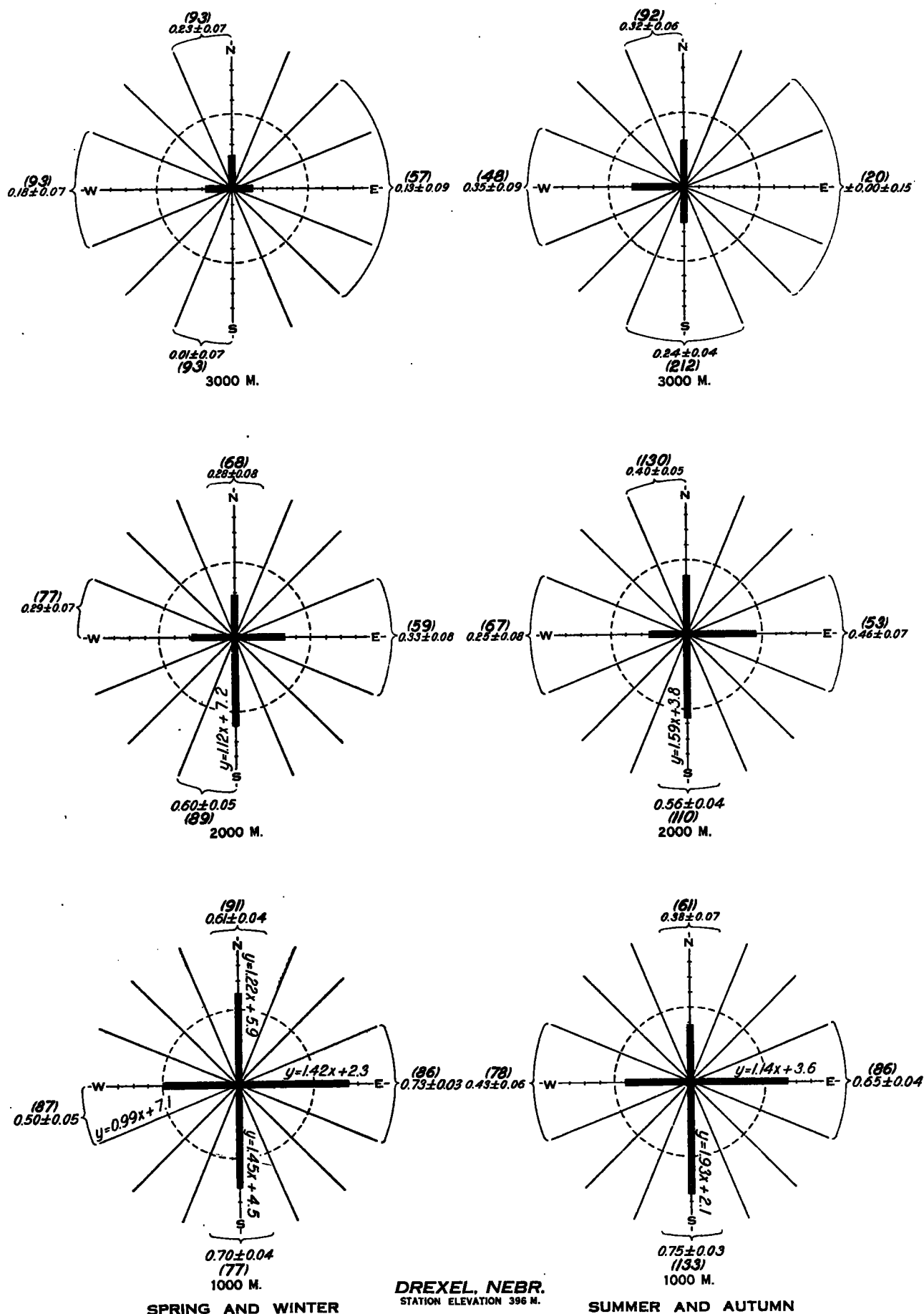


FIG. 1.—Correlation coefficients for surface-wind velocities and those in the free air at heights of 1,000, 2,000, and 3,000 meters above sea level, at Drexel, Nebr., with related data.

tial end desired, viz, the possibility of readily computing the velocity aloft when only the surface velocity and direction are known. In the other case it would be necessary to know the surrounding surface-pressure distribution.

The spring and winter seasons and summer and autumn seasons were taken together, first, because of the insufficient number of observations in any single season, and, second, because of the seasonal lag in velocities as shown in the means, autumn velocities being less than those in the spring.

The midway direction through which the surface wind turned during the kite flight was taken as the surface direction in this classification. Pilot-balloon records were not included in this study, but it is hoped to incorporate these data in a similar study some time in the future.

The correlation coefficients were computed by the usual formula, viz.

$$r = \frac{\sum(xy)}{n\sigma_x\sigma_y}, \quad (1)$$

in which the  $x$ 's are the series of deviations from the arithmetic mean of one series and the  $y$ 's the corresponding deviations from the arithmetic mean in the other series. In this case the surface velocities correspond to the  $x$ 's and the upper velocities to the  $y$ 's.

In each case where the correlation coefficient was found to be 0.50 or higher with a relatively small probable error, the regression equation was determined. This equation is of the type,  $y = bx$ ; the regression coefficients are determined by the equations,

$$b_1 = r \frac{\sigma_x}{\sigma_y}, \quad (2)$$

$$\text{and} \quad b_2 = r \frac{\sigma_y}{\sigma_x}, \quad (3)$$

respectively. The equation of the straight line defined by the coefficient of correlation and expressing the direct relation of the surface and the upper wind velocity is<sup>7</sup> where

$$y = r \frac{\sigma_y}{\sigma_x} x, \quad (4)$$

$r$ , is the correlation coefficient,  $x$ , the surface wind velocity,  $y$ , the upper velocity.  $\sigma_x$  and  $\sigma_y$ , the standard deviations of the surface and upper wind velocities with respect to their arithmetic means, respectively. For practical purposes as explained by Yule<sup>8</sup> it is more convenient to express the equations in terms of the absolute values of the variables rather than the deviations; therefore replacing  $x$  by ( $X$  minus the arithmetic mean of  $x$ , for each particular group of  $x$ 's) and  $y$  by ( $Y$  minus the arithmetic mean of  $y$ , for each particular group of  $y$ 's), thereby changing the regression equation,  $y = bx$ , to the ordinary form of an equation for a straight line,  $y = bx + a$ .

As an illustration, let us take the south surface winds for spring and winter at Drexel, Nebr. Let it be desired to obtain an equation whereby the wind velocity at the height of 1,000 meters (M. S. L.) may be computed from the surface velocity when the wind direction is from the south. The velocities at the surface and the corresponding velocities at 1,000 meters (M. S. L.) are tabulated in Table I, the surface velocities being designated  $x$  and the upper velocities  $y$ . The departures of each value of  $x$  and  $y$  are then found from their respective arithmetic means and are tabulated under the columns headed A and B, respectively.

Thus we have when substituting in equation (1),

$$r = \frac{811.04}{77 \times 2.70 \times 5.60} = 0.70$$

as the value of the correlation coefficient for these data.

<sup>7</sup> Marvin, C. F. MO. WEATHER REV. October, 1916, 44: 567.  
<sup>8</sup> Yule, G. U.: *Introduction to the theory of statistics* p. 179. 1916.

TABLE I.—Computation of correlation coefficient for wind velocities at the surface and those at 1,000 meters above sea level at Drexel, Nebr. (surface direction, south) spring and winter.

Surface velocity.	1,000 meter velocity.	$x$ Surface velocity.	$y$ 1,000 meter velocity.	$x$ Surface velocity.	$y$ 1,000 meter velocity.	A.	A.	A.	B.	B.	B.	A :	A :	A :	B :	B :	B :	A × B.	A × B.	A × B.
4	7	6	8	5	11	-2.8	-0.8	-1.8	-7.4	-6.4	-3.4	784	64	324	5,476	4,096	1,156	+2,072	+512	+612
6	12	5	9	6	16	-0.8	-1.8	-0.8	-2.4	-5.4	-1.6	64	324	64	576	2,916	256	+192	+972	-128
9	14	6	15	8	21	+2.2	-0.8	+1.2	-0.4	+0.6	+6.6	484	64	144	16	36	4,356	-88	-48	+792
10	18	9	21	5	16	+3.2	+2.2	-1.8	+3.6	+6.6	+1.6	1,024	484	324	1,296	4,356	256	+1,152	+1,452	-288
7	12	10	21	7	14	+0.2	+3.2	+0.2	-2.4	+6.6	-0.4	4	1,024	4	576	4,356	16	-48	+2,112	-8
5	8	4	11	5	17	-1.8	-2.8	-1.8	-6.4	-3.4	+2.6	324	784	324	4,096	1,156	676	+1,152	+952	-468
9	25	6	9	6	14	+2.2	-0.8	-0.8	+10.2	-5.4	-0.4	484	64	64	11,236	2,916	16	+2,332	+432	+32
5	11	3	10	14	24	-1.8	-3.8	+7.2	-3.4	-4.4	+9.6	324	1,444	5,184	1,156	1,636	9,216	+612	+1,672	+6,912
5	9	5	20	5	7	-1.8	-1.8	-1.8	-5.4	+5.6	-7.4	324	324	324	2,916	3,136	5,476	+972	-1,008	+1,332
5	9	4	10	13	24	-1.8	-2.8	+6.2	-5.4	-4.4	+9.6	324	784	3,844	2,916	1,636	9,216	+972	+1,232	+5,952
9	19	9	20	9	19	+2.2	+2.2	+2.2	+4.6	+5.6	+4.6	484	484	484	2,116	3,136	2,116	+1,012	+1,232	+1,012
9	21	7	14	5	9	+2.2	+0.2	-1.8	+6.6	-0.4	-5.4	484	4	324	4,356	16	2,916	+1,452	-48	-972
13	21	5	12	4	7	+6.2	-1.8	-2.8	+6.6	-2.4	-7.4	484	324	784	4,356	576	5,476	+4,092	+432	+2,072
8	11	4	9	8	14	+1.2	-2.8	+1.2	-3.4	-5.4	-0.4	3,844	784	144	1,156	2,916	16	-408	+1,612	-48
7	13	4	7	5	7	+0.2	-2.8	-1.8	-1.4	-7.4	-7.4	4	784	324	196	5,476	5,476	-28	+2,072	+1,332
10	17	5	12	6	12	+3.2	-1.8	-0.8	+2.6	-2.4	-2.4	1,024	324	64	676	576	576	+832	+432	+192
5	9	7	19	6	13	-1.8	+0.2	-0.8	-5.4	+4.6	-1.4	324	4	64	2,916	2,116	196	+972	+92	+112
6	12	4	17	11	21	+0.2	-2.8	+4.2	+4.6	+2.6	+6.6	4	784	1,764	2,116	676	4,356	+92	-728	+2,772
18	23	4	15	9	21	-0.8	+0.2	+2.2	-2.4	+3.6	+6.6	64	4	484	576	1,296	4,356	+192	+72	+1,452
5	7	6	16	4	9	+11.2	-2.8	+2.2	+8.6	+0.6	+2.6	12,544	784	484	7,396	36	676	+9,632	-168	+572
7	12	4	7	5	21	-1.8	-0.8	-2.8	-7.4	+1.6	-5.4	324	64	784	5,476	256	2,916	+1,332	-128	+1,188
6	14	8	22	5	3	-0.8	+1.2	-1.8	-0.4	+7.6	-11.4	64	144	324	16	5,776	12,996	+32	+912	+2,062
11	18	8	30	6	16	+4.2	+1.2	-0.8	+3.6	+15.6	+1.6	1,764	144	64	1,296	24,336	256	+1,512	+1,872	-128
10	23	8	13	4	10	+3.2	+1.2	-2.8	+8.6	-1.4	-4.4	1,024	144	784	7,396	196	1,936	+2,752	-168	+1,232
7	7	4	7	7	-----	+0.2	-2.8	-----	-7.4	-7.4	-----	4	784	-----	5,476	5,476	-----	-148	+2,072	-----
Total.	-----	-----	-----	525	1,106	-----	-----	-----	-----	-----	-----	-----	-----	560.88	-----	-----	2,407.92	-----	-----	+811.04
Means.	-----	-----	-----	0.8	14.4	-----	-----	-----	-----	-----	-----	-----	-----	7.28	-----	-----	31.27	-----	-----	-----
Sq. rts.	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	2.70	-----	-----	5.60	-----	-----	-----

$$r = \frac{\sum(xy)}{n\sigma_x\sigma_y} = \frac{811.04}{77 \times 2.70 \times 5.60} = 0.70.$$

$$P. E. = 0.674 \frac{1-r}{\sqrt{n}} = 0.674 \frac{1-0.70}{\sqrt{77}} = \pm 0.04.$$

The probable error of this coefficient is,

$$P. E. = 0.674 \frac{1-r^2}{\sqrt{n}} \quad (5)$$

where,  $n$  is the number of observations upon which it is based. Substituting in formula (5), we have,

$$P. E. = 0.674 \frac{1-.4900}{\sqrt{77}} = \pm 0.04$$

Since it is desired to compute the upper wind velocity or  $y$ , in terms of the surface velocity or  $x$ , equation (3) is used.<sup>9</sup>

Therefore the regression equation  $y = bx$ , becomes

$$y = r \frac{\sigma_y}{\sigma_x} x, \quad (4)$$

Substituting the computed values from Table I in equation (4), we have

$$y = 0.70 \frac{5.60}{2.70} x \text{ or} \\ y = 1.45x \quad (6)$$

But in order to obtain an equation of a straight line in the usual form,<sup>10</sup> i. e.,  $y = bx + a$ , we replace  $x$  in equation (6) by  $X - 6.8$  (the arithmetic mean of the surface winds) and  $y$  by  $Y - 14.4$  (the arithmetic mean of the upper winds), as found in Table I. Then substituting in equation (6) we have,

$$Y = 1.45X + 4.5 \quad (7)$$

It must be understood, of course, that the law of relation represented by the equation is purely an arbitrary one and applies only to conditions within the range of the records discussed. The degree of dependence which may be given the results obtained by these equations will be discussed later.

The above procedure has been carried out in a like manner for other stations and cases, but, as already stated, the regression coefficients have been found only when the coefficient of correlation was 0.50 or higher, with a small probable error, as useful results could not be expected where only a poor or no correlation existed.

Figure 1 shows these results graphically for Drexel, Nebr., and a little study of this chart will make comparison simple. In order to assist in this respect, superposed bars, with lengths proportional to the correlation coefficient, have been drawn to visualize the degree of correlation for the various altitudes and surface directions.

The correlation coefficients together with their probable errors are shown adjacent to the surface direction or group of directions to which they apply.

Regression equations (in which  $x$  = surface velocity and  $y$  = velocity aloft) for computing the upper wind velocities are shown along with the correlation coefficient whenever the latter has a significantly high value of 0.50 or more.

The figures in parentheses indicate the number of observations upon which the correlation coefficient is based.

Lack of space prohibited showing these data graphically for the other stations, but the data themselves are given in tabular form in Table 2. The column headings in this table are explained as follows: Column headed "wind direction" gives the surface direction for which the correlation coefficient was computed. In cases where

more than one direction occurs, an insufficient number of observations from the cardinal direction, i. e., N., S., E., or W., was available and the adjacent directions were therefore included; thus, extreme directions only of such groupings are indicated;  $n$  indicates the number of observations upon which the correlation coefficient is based,  $r$  shows the correlation coefficient and its probable error, and  $y$  represents the wind velocity aloft and is that value found by the regression equation shown in this column,  $x$  representing the surface wind velocity. No regression equation is shown for correlation coefficients less than 0.50, as previously explained.

Figure 3 shows how these computations may be obtained graphically by drawing the regression line on coordinate paper. The line in the figure is that for south surface winds at Drexel, Nebr., for spring and winter when surface velocities were correlated with those at 1,000 meters above sea level. For example, to find the upper velocity with a surface velocity of 8.7 m. p. s. from the south we find 8.7 on the horizontal scale and run up to its intersection with the regression line and read across on the vertical scale 17.1 m.p.s., the same as is obtained when 8.7 is substituted in the regression equation  $y = 1.45x + 4.5$ .

It will be noticed that for Drexel and Ellendale the 1,000 meter (M. S. L.) level was chosen while for the other stations, the 750 meter (M. S. L.) level was taken. This was done because of the difference in elevation above sea-level of the stations themselves. Thus for the three lower stations the 750 meter (M. S. L.) level is practically the same height above the surface as the 1,000 meter (M. S. L.) level is above the surface at Drexel and Ellendale, the higher stations.

A few remarks regarding the interpretation and significance of correlation coefficients and their reliability as measures of relationship seem appropriate.

The function of the correlation coefficient is that of an index of the extent to which the relation between certain data may be represented by a straight line. A low correlation coefficient must not necessarily be interpreted to mean no relation, but that if a relation exists it is either small or is not well represented by a straight line.<sup>11</sup> The value by the very nature of the equation itself can never be greater than unity, i. e., +1 and -1, indicating perfect correlation, positive in the first case and negative or inverse in the second, while 0 indicates no correlation. The following rules will assist in giving a general idea of the interpretation of the correlation coefficient,  $r$ , according to its relation to its probable error.

1. If  $r$  is less than the probable error, there is no evidence whatever of relation.

2. If  $r$  is more than 6 times the probable error, the existence of correlation is a practical certainty.

3. When the probable error is relatively small a value for  $r$  of less than 0.30 can not be considered to indicate correlation at all marked, while a value of more than 0.50 is evidence of good correlation. Mr. W. H. Dines, of the British Meteorological Office, points out, however, that in forecasting one variable in terms of the other, too much dependence should not be placed on a correlation coefficient as low as 0.50, even though the probable error is relatively small. He shows that, "If there is a cause  $A$  and a result  $M$  with a correlation  $r$  between them, then in the long run  $A$  is responsible for  $r^2$  of the variation of  $M$ ."<sup>12</sup> Obviously, then, on the average, with a

<sup>9</sup> Marvin, C. F., *loc. cit.*

<sup>10</sup> Yule, G. V., *loc. cit.*

<sup>11</sup> Marvin, C. F. *MO. WEATHER REV.* October, 1916, 44:560.

<sup>12</sup> *Meteorological Magazine*, February, 1921, p. 20.

correlation of 0.50, only 25 per cent, i. e.  $(0.50)^2$ , of the causes of the changes in the variable can be attributed to the correlated associate.

The striking feature shown in Figure 1 for Drexel and in Table 2 for all stations is the generally high correlation existing between the velocities at the surface and those between 500 and 600 meters above. The coefficients indicate a somewhat better relation during the spring and winter than during the summer and autumn. Groes-

The correlation coefficients for 3,000 meters were computed for Drexel and Ellendale, there being too few observations at the other stations for this purpose. No values as high as 0.50 were found, but the west surface winds at Ellendale for spring and winter gave a surprisingly high value of  $0.44 \pm 0.08$ . A significant feature of nearly all the values for this level was the extremely low degree of correlation, some directions indicating zero while others showed a small negative value.

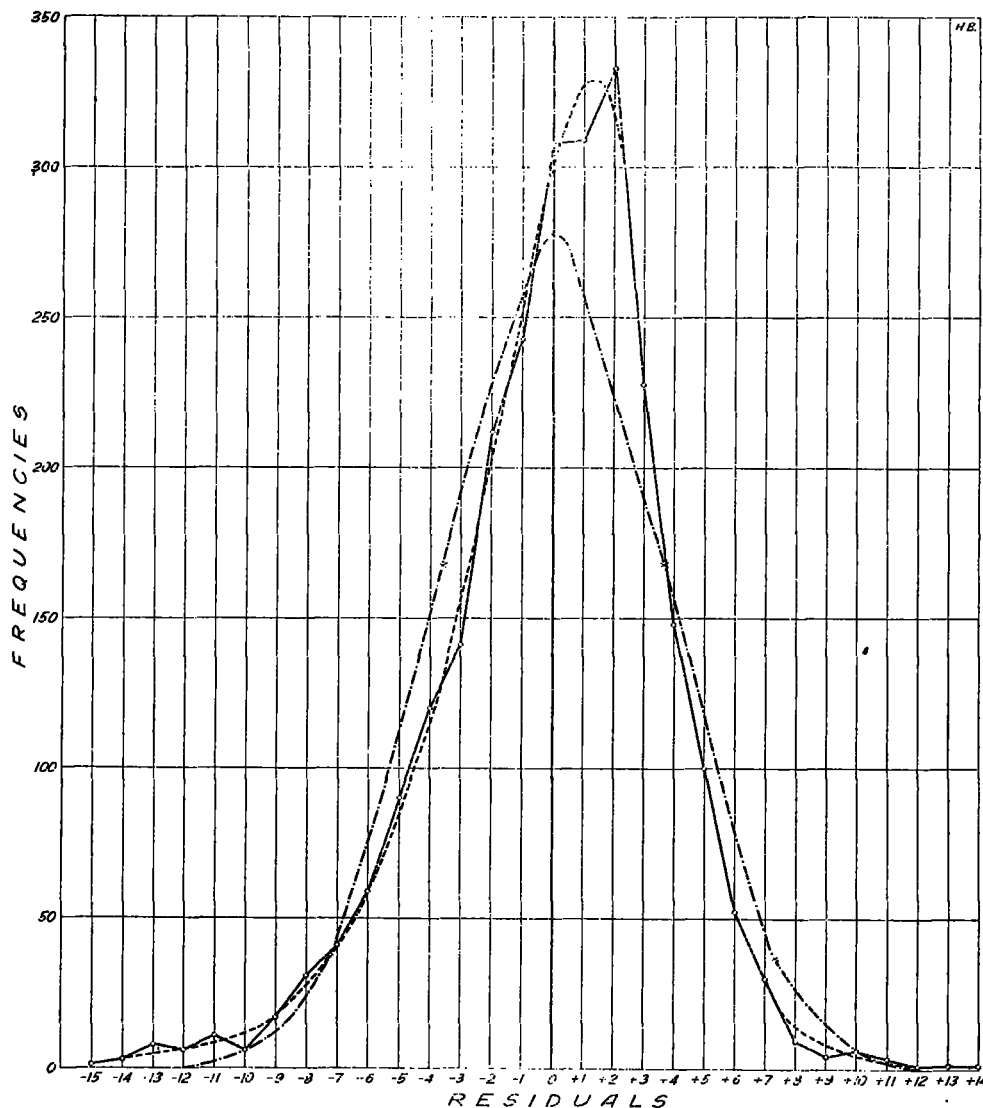


FIG. 2.—Frequency polygon showing the distribution of the residuals. m. p. s. (computed minus observed velocities).

beck, the station farthest south, shows the least correlation for this level, the coefficients being less than 0.50 for all directions, except north for both periods and west for spring and winter.

The 2,000 meter level does not indicate nearly as good correlation as does the lower level. This is to be expected, since the wind at the higher levels becomes more and more influenced by other factors, which in turn do not influence the winds at the surface to such a great extent. The south surface winds at Drexel for this level maintain high correlation coefficients for both periods, and the same is true for these winds at Broken Arrow for the spring and winter. Groesbeck has a significant value for north and west surface winds for spring and winter for this level.

#### PROBABLE ACCURACY TO BE EXPECTED.

When using equations such as these there is always a degree of uncertainty as to the correctness of the computed result in individual cases. Statistics furnish various methods of determining the probability within certain limits of accuracy when the data fall in the category of normal distributions. Attempt will be made to show to what extent this is true in our study.

The wind aloft was computed from the surface velocity by the regression equation whenever the latter was determined. The residuals of these computed velocities were then found by subtracting the observed from the computed. These residuals for the winds of the various directions were then charted as a histogram, and it was

found that the different groups were nearly identical as to distribution. The data were therefore regarded as homogenous and a single group was made of all. Figure 2 is the resulting frequency polygon.

The great number of observations (2,522) included in this graph justifies its acceptance as typical and representative. It is apparent that the distribution is not symmetrical. Nevertheless, the normal curve of best fit to the data is shown on the drawing by the dot-and-dash line. Since the maximum height of this normal

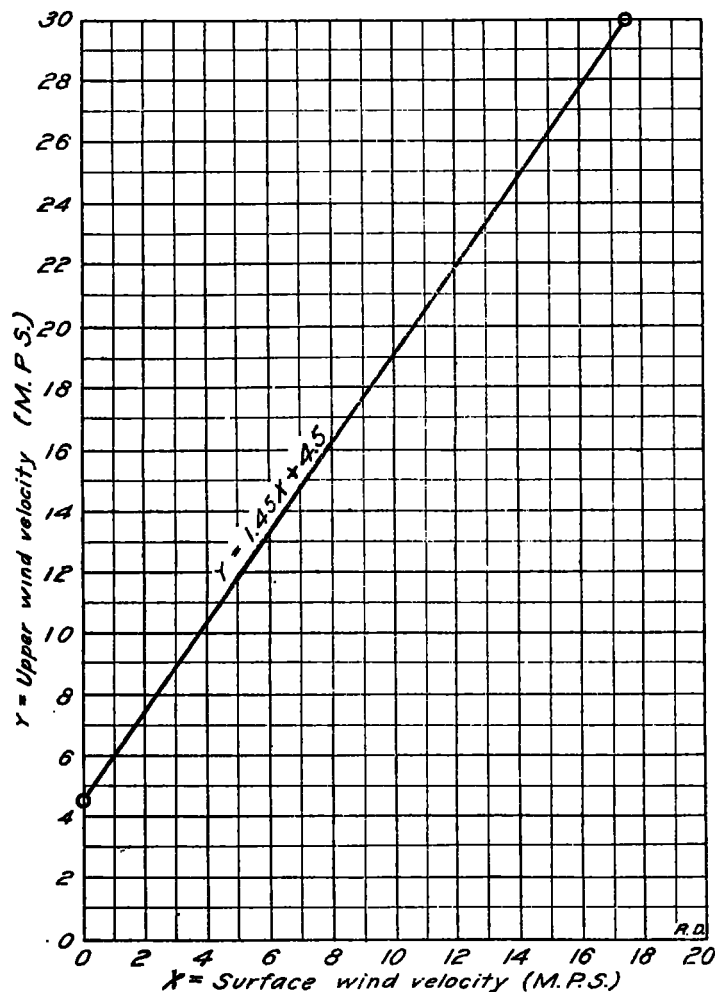


FIG. 3.—Example of graphical representation of regression equation.

curve and the actual data are nearly equal we may conclude that the data are comparatively elemental, although slightly askew.

In addition, it has seemed sufficient to draw by free-hand a smooth asymmetrical curve fitting the observed data as closely as possible, in order to show the probability of residuals of different magnitude.

The significant fact brought out by this frequency polygon is the large proportion of computed velocities with small residuals and the extremely small percentage having large residuals.

The following values from these residuals were computed: Arithmetic mean (disregarding signs), 2.8, median, zero class, mode, +2 class. It was found that the sums of all plus and minus residuals were nearly equal, viz, +3557 and -3524, respectively. This small difference seems negligible. The number of plus residuals disregarding their size show a decided preponderance, amounting to 55 per cent of the total. This is apparent from the skewness exhibited in the graph. The reason for this skewness is not certain but seems probably to be a result of the distribution of the observed upper wind velocities with respect to their arithmetic means. These winds distribute themselves as do a number of meteorological phenomena, such as rainfall, temperature, etc., with a preponderance of values less than the mean. This distribution is to be expected in such cases, since a few extremely large plus departures necessarily tend to raise the value of the mean while it is impossible to have any negative departures of greater magnitude than the mean, thereby causing a preponderance of negative values. It was found that 55 per cent of the departures of the observed wind velocities with respect to their means were negative and since the residuals shown graphically in Figure 2 were found by subtracting the observed from the computed velocities, it seems likely that an equal preponderance should result, viz, 55 per cent plus residuals, or similar to what was found. This is offered merely as a possible reason of this skewness but the true explanation is not certain.

After pilot balloon records have been incorporated into this study and the number of stations consequently increased, it will be possible to establish zones in which these equations will be applicable; but until this is done, the use of the equations is limited to the general vicinity of the five kite stations, except in so far as interpolation is possible between them.

#### CONCLUSION.

It appears from the above discussion to be entirely practicable to expect reasonably accurate results in computing wind velocities aloft by the regression equations for those directions indicating a significant correlation coefficient. If results are desired in miles per hour, the conversions should be made after all the computations have been made using meters per second.

It seems possible that, when pilot-balloon observations are impracticable, a good estimate of the upper winds may be obtained by the use of these equations. These computations are performed by substituting the surface wind velocity (m. p. s.) for  $x$ , in the particular equation for the prevailing surface direction, and solving for  $y$ , the velocity aloft. These equations for their respective directions are given in Table 2. The erratic conditions found aloft with certain winds is fully realized but with an increasing amount of observational data available, the application of statistical treatment should prove of practical value in many ways.

Acknowledgment is due Mr. Edgar W. Woolard for information regarding statistical methods and to the employees of the Aerological Division for assistance rendered in checking the large amount of computational work.

TABLE 2.—Correlation coefficients and related data for various stations and elevations.

## DREXEL, NEBR. (396 METERS ABOVE SEA-LEVEL).

Season.	1,000 meters (M. S. L.).				2,000 meters (M. S. L.).				3,000 meters (M. S. L.).		
	Surface wind direction.	n.	r.	y.	Surface wind direction.	n.	r.	y.	Surface wind direction.	n.	r.
Spring and winter.....	n.....	91	0.61±0.04	1.22r+5.9	n.....	68	0.28±0.08	.....	nnw.-n.....	93	0.23±0.07
	ene.-ese.....	86	.73±.03	1.42r+2.3	ene.-ese.....	59	.33±.08	.....	ne.-se.....	57	.13±.09
	s.....	77	.70±.04	1.45r+4.5	s.-ssw.....	89	.60±.05	1.12r+7.2	s.-ssw.....	93	.01±.07
	ws.-w.....	87	.50±.05	.96r+7.1	w.-wnw.....	77	.29±.07	.....	ws.-wnw.....	93	.18±.07
Summer and autumn.....	n.....	61	.38±.07	.....	nnw.-n.....	130	.40±.05	.....	nnw.-n.....	92	.32±.08
	ene.-ese.....	86	.65±.04	1.14r+3.6	ene.-ese.....	53	.48±.07	.....	ne.-se.....	20	.00±.15
	s.....	133	.75±.03	1.93r+2.1	s.....	110	.56±.04	1.59r+3.8	s.-ssw.....	212	.24±.04
	ws.-wnw.....	78	.43±.06	.....	ws.-wnw.....	67	.25±.08	.....	ws.-wnw.....	48	.35±.09

## ELLENDALE, N. DAK. (444 METERS ABOVE SEA LEVEL).

Spring and winter.....	n.-nne.....	85	0.79±0.03	0.85r+2.8	nnw.-n.....	79	0.35±0.07	.....	nnw.-nne.....	51	0.39±0.08
	ne.-se.....	71	.50±.06	.75r+4.4	ne.-se.....	51	.18±.09	.....	ne.-se.....	22	.12±.14
	s.-ssw.....	103	.60±.01	1.05r+4.4	s.-ssw.....	85	.29±.07	.....	s.-ssw.....	63	.02±.08
	ws.-wnw.....	59	.60±.06	1.11r+5.6	sw.-wnw.....	70	.45±.06	.....	ws.-wnw.....	41	.44±.10
Summer and autumn.....	n.-nne.....	90	.68±.04	1.03r+2.2	nnw.-nne.....	78	.48±.06	.....	nnw.-nne.....	42	.14±.10
	ne.-se.....	105	.47±.05	.....	ne.-se.....	73	.28±.07	.....	ne.-se.....	31	.24±.11
	s.....	109	.61±.04	1.26r+3.4	s.-ssw.....	96	.47±.05	.....	s.-ssw.....	78	.18±.07
	w.-wnw.....	73	.67±.04	1.21r+4.0	ws.-wnw.....	83	.38±.06	.....	ws.-wnw.....	64	.26±.08

## BROKEN ARROW, OKLA. (233 METERS ABOVE SEA LEVEL).

Season.	750 meters (M. S. L.).				2,000 meters (M. S. L.).			
	Surface wind direction.	n.	r.	y.	Surface wind direction.	n.	r.	y.
Spring and winter.....	nnw.-nne.....	70	0.60±0.05	0.92r+2.4	nnw.-nne.....	38	0.32±0.10	.....
	ne.-se.....	42	.48±.08	.....	ne.-se.....	23	.19±.14	.....
	s.-ssw.....	132	.65±.03	1.13r+4.4	s.-ssw.....	60	.52±.06	0.99r+4.1
	sw.-wnw.....	50	.52±.07	1.41r+1.3	sw.-nw.....	38	.10±.11	.....
Summer and autumn.....	nnw.-nne.....	58	.57±.06	.89r+3.1	nnw.-nne.....	26	.42±.11	.....
	ne.-se.....	49	.59±.06	.95r+3.7	ne.-se.....	27	.43±.11	.....
	s.....	94	.57±.05	.95r+4.2	s.-ssw.....	98	.43±.06	.....
	sw.-wnw.....	33	.46±.09	.....	sw.-nw.....	28	.27±.12	.....

## GROESBECK, TEX. (141 METERS ABOVE SEA LEVEL).

Spring and winter.....	nnw.-nne.....	93	0.68±0.04	1.09r+2.6	nw.-nne.....	72	0.54±0.06	1.10r+5.7
	ne.-se.....	68	.31±.07	.....	ne.-se.....	38	.09±.11	.....
	s.-ssw.....	124	.43±.05	.....	s.-ssw.....	89	.20±.07	.....
	sw.-nw.....	68	.55±.06	.77r+7.1	sw.-nw.....	55	.60±.06	1.12r+7.1
Summer and autumn.....	nnw.-nne.....	61	.52±.06	.96r+3.5	nw.-nne.....	38	.36±.10	.....
	ne.-se.....	78	.42±.06	.....	ne.-se.....	41	.39±.09	.....
	s.-ssw.....	124	.46±.05	.....	s.-ssw.....	99	.29±.06	.....
	sw.-nw.....	58	.37±.03	.....	sw.-nw.....	42	.30±.09	.....

## ROYAL CENTER, IND. (225 METERS ABOVE SEA LEVEL).

Spring and winter.....	nnw.-nne.....	25	0.70±0.07	1.21r+1.4	Too few observations.	.....	.....	.....
	ne.-se.....	65	.47±.07	.....	ne.-se.....	34	0.19±0.11	.....
	s.-ssw.....	59	.59±.06	1.48r+5.1	s.-ssw.....	50	.36±.08	.....
	ws.-wnw.....	89	.67±.04	.93r+4.4	ws.-wnw.....	72	.30±.07	.....
Summer and autumn.....	nnw.-nne.....	35	.49±.09	.....	Too few observations.	.....	.....	.....
	ne.-se.....	59	.53±.06	1.08r+2.5	ne.-se.....	24	.23±.13	.....
	s.-ssw.....	75	.66±.04	1.48r+2.8	s.-ssw.....	54	.36±.08	.....
	ws.-wnw.....	99	.45±.05	.....	ws.-wnw.....	81	.28±.07	.....